INSTRUCTIONAL VIDEO:

Binary Numbers

“10b, || ~10b: that is the question:...”
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Learning Objectives

Abstractions & Modularity
Learning Objectives

Advanced Placement Computer Science Principles
► Big Idea 2: Abstractions
  ❑ 2.1.1 - Describe the variety of abstractions used to represent data.
  ❑ 2.1.2 - Explain how binary sequences are used to represent digital data.
  ❑ 2.2.3 - Identify multiple levels of abstractions that are used when writing programs.
Learning Objectives

Advanced Placement Computer Science A

- Big Idea 1: Modularity
  - Modularity requires students to simplify concepts and processes by looking at the big picture rather than the details and developing abstractions. Whether this is in the representation of objects or concepts, in the use of preexisting processes, or in the creation and organization of code into different methods or classes. There are several instructional strategies that can help students make these connections, including using manipulatives and diagramming.
Number Systems

As human civilizations arose, people needed to find more efficient and effective ways of completing necessary tasks, such as...

- Tallying items
- Recording data
- Representing information
- Conducting business transactions
- Solving problems
- Sharing items
- ...and many more.
Thus, number sets were created.

- **Natural Numbers (counting numbers)**: 1, 2, 3, 4, 5, 6, 7, ...
- **Whole Numbers**: 0, 1, 2, 3, 4, 5, 6, 7, ...
- **Integer Numbers**: ..., -3, -2, -1, 0, 1, 2, 3, ...
- **Rational Numbers**: Numbers which can be represented as a terminating or repeating decimal, like \( \frac{1}{2} \), 7.5, or 3.3.
- **Irrational Numbers**: Numbers which cannot be represented as a terminating or repeating decimal, like \( \pi \), e, or \( \sqrt{2} \).
- **Real Numbers**: The union of Rational and Irrational Numbers.
- **Imaginary Numbers**: The form \( bi \) where \( b \) is a real number and \( i = \sqrt{-1} \).
- **Complex Numbers**: The form \( a + bi \) where \( a \) and \( b \) are real numbers.
These previously described number sets utilized the Arabic numerals, or digits:

0, 1, 2, 3, 4, 5, 6, 7, 8, 9

This was handy because one could easily count using one’s fingers.

The digits’ grouping and position in the whole value made for more robust decimal, or base-10, numbers.

For example, 352 could be represented by the digit 3 in the hundreds place, the digit 5 in the tens place and the digit 2 in the ones place. Or,…

\[3 \times 100 + 5 \times 10 + 2 \times 1\]

or

\[3 \times 10^2 + 5 \times 10^1 + 2 \times 10^0\]
Number Systems (continued)

If you interact with computers long enough, you’ll see other types of number systems as well, such as...

- **Binary Number System**, the most basic encoding type:
  
  0, 1

- **Octal Number System**

  0, 1, 2, 3, 4, 5, 6, 7

- **Hexadecimal Number System**

  0, 1, 2, 3, 4, 5, 6, 7, 8, 9, a, b, c, d, e, f
Why Do Computers Use Binary Numbers?

We are going to get to the 0’s and 1’s of the matter.
Why Do Computers Use Binary Numbers?

Did you know that the word “computer” was first used to describe humans and not machines?

- In 1613, a computer was a person who performed calculations or computations.
- During the rise of the Industrial Revolution, machines were created whose main purpose was to perform calculations and computations. These machines were then called computers.
- In the 1930’s, the digital computer began development.
- These devices needed a simple way to process data.
- Similar to using Morse Code (dots and dashes) to encode messages or using drumming (the low and high tones) to communicate, the principle of binary was implemented.
Why Do Computers Use Binary Numbers? (continued)

Binary Number System

- Number system which uses only two distinct states (0 and 1) called binary digits, or bits.
- In a sequence of bits, each bit is represented by an exponent of base-2 in descending order of degree. Therefore, a sequence of eight bits, or one byte, can represent 256 different permutations for data, such as numbers, characters, etc.
- Transistors in computer chips act as switches to indicate whether a bit is off (0 represents off) or on (1 represents on).
- It allows for more manageable storage of bits of data.
- It’s easier to transmit bits of data from one device to another.
- It is less expensive to create components that read two states instead of more complex ones that need to be able to read two and then varying states in between.
Converting Between Binary And Decimal Number Systems

“There are only 10 types of people in the world: those who understand binary and those who don’t.”
Converting Between Binary And Decimal Number Systems

From Decimal to Binary

- Given a decimal number, like 173
- Divide the decimal number by the highest exponent of 2 that’s equal to or less than the decimal number. That quotient becomes your leading binary digit, meaning that the corresponding bit is turned on.
  
  \[ 173 / 2^7 \text{ (or 128)} = 1 \text{ with a remainder of 45} \]

- The remainder becomes your sub-decimal value. Divide this new value by the next descending exponent of 2. If the quotient is 1, turn that bit on. If it is 0, turn that bit off.

  \[ 45 / 2^6 \text{ (or 64)} = 0 \text{ with a remainder of 45} \]

- Repeat the previous step until the remainder is 0. All subsequent bits will also be 0.

  \[ 45 / 2^5 \text{ (or 32)} = 1 \text{ with a remainder of 13} \]
Converting Between Binary And Decimal Number Systems (continued)

From Decimal to Binary (continued)

\[
\begin{align*}
13 / 2^4 \text{ (or 16)} &= 0 \text{ with a remainder of 13} \\
13 / 2^3 \text{ (or 8)} &= 1 \text{ with a remainder of 5} \\
5 / 2^2 \text{ (or 4)} &= 1 \text{ with a remainder of 1} \\
1 / 2^1 \text{ (or 2)} &= 0 \text{ with a remainder of 1} \\
1 / 2^0 \text{ (or 1)} &= 1 \text{ with a remainder of 0}
\end{align*}
\]

Thus, the decimal value 173 has a binary equivalent of 10101101.
Converting Between Binary And Decimal Number Systems (continued)

From Decimal to Binary (continued)

► Now you try.

► Example 1: Please convert the following decimal value to it’s corresponding binary value.

121
Converting Between Binary And Decimal Number Systems (continued)

From Decimal to Binary (continued)

► Example 1: Please convert the following decimal value to its corresponding binary value.

121

► Solution:

121 / 2^7 = 0 with a remainder of 121
121 / 2^6 = 1 with a remainder of 57
57 / 2^5 = 1 with a remainder of 25
25 / 2^4 = 1 with a remainder of 9
9 / 2^3 = 1 with a remainder of 1
1 / 2^2 = 0 with a remainder of 1
1 / 2^1 = 0 with a remainder of 1
1 / 2^0 = 1 with a remainder of 0

Thus the binary equivalent to 121 is 0111001, or 111001.

► When considering significant digits, leading zeros can be dropped.
From Binary to Decimal

- Just like with decimal numbers, you can look at binary numbers in terms of place value.
- Consider the binary number 11111110.

\[
\begin{array}{cccccccc}
1 & 1 & 1 & 1 & 1 & 1 & 0 \\
2^7 & 2^6 & 2^5 & 2^4 & 2^3 & 2^2 & 2^0 \\
128 & 64 & 32 & 16 & 8 & 4 & 2 \\
\end{array}
\]

- There is 1 bit turned on in the 128’s place, 64’s place, 32’s place, 16’s place, 8’s place, 4’s place, and 2’s place.
- Just find the sum of all the bits’ place values that are turned on, ie.

\[
128 + 64 + 32 + 16 + 8 + 4 + 2 = 254
\]
- Thus, the decimal number equivalent to the binary number 11111110 is 254.
Example 2: Please convert the following binary value to its corresponding decimal value.

110011
Converting Between Binary And Decimal Number Systems (continued)

From Binary to Decimal (continued)

► Example 2: Please convert the following binary value to its corresponding decimal value.

► Solution:

```
  1 1 0 0 1 1
2^7 2^6 2^5 2^4 2^3 2^2 2^1 2^0
128  64  32  16   8   4   2   1
```

\[ 32 + 16 + 2 + 1 = 51 \]

Thus, the decimal number equivalent to the binary number 110011 is 51.
Summary

The sum of 100 and 100 is 1000.
Number Systems

- Human
  - Decimal: Base-10

- Computer
  - Binary: Base-2
  - Octal: Base-8
  - Hexadecimal: Base-16

Why Do Computers Use Binary Numbers?

- Represents 2 distinct states like an electrical switch
- Easier to manipulate data
- More efficient for storage
- Less expensive to use

Converting Between Binary And Decimal Number Systems

- Decimal to Binary - Divide by largest degree of base 2 that’s equal to or less than base-10 value, and subsequent remainder values, to turn corresponding bit on and repeat.
- Binary to Decimal - Find the sum of all turned on bits.
Resources

Helpful Links.
Resources

- Hex, Decimal, Octal, Binary Converter
- How To Create The Matrix Rain In Command Prompt
- Number Systems
  - https://www.robotroom.com/NumberSystems.html
- Number Systems: An Introduction to Binary, Hexadecimal, and More
  - https://code.tutsplus.com/articles/number-systems-an-introduction-to-binary-hexadecimal-and-more--active-10848
- Numeral System and Its Importance
Resources (continued)

- Pioneers @ KerryR.net
  - http://www.kerryr.net/pioneers/shannon.htm
- When Was The First Computer Invented?
  - https://www.computerhope.com/issues/ch000984.htm
- Why Binary Numbers Are Used By Computers?
- Why Is Binary Used In Electronics And Computers?
Contact
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